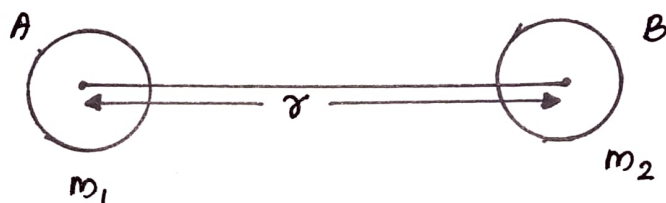


GRAVITATION

Newtons Universal Law of Gravitation

Every Particle of matter in the universe attracts every other particles with a force which is directly proportional to the product of their masses and inversely proportional to the square of distance between them.



If m_1 and m_2 are the masses of the two particles A and B separated by a distance ' r ' then the force of attraction,

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = \frac{G m_1 m_2}{r^2}$$

where ' G ' is the gravitational Constant

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

If $m_1 = m_2 = 1 \text{ kg}$ and $r = 1 \text{ m}$

then $G = F$

Hence gravitational Constant is numerically equal to force of attraction between two unit masses separated by unit distance.

Thus gravitational force

1. it is always attractive
2. it is independent of the intervening medium
3. it is an action-reaction pair
4. it is a two body interaction
5. The gravitational force is Conservative
6. It holds good over a wide range of distances.

Acceleration due to gravity (g)

The uniform acceleration produced in a freely falling body due to gravitational pull of the earth is known as acceleration due to gravity.

Let us consider the earth to be a sphere of Radius 'R' and mass 'M'. Consider a body of mass 'm' placed on the surface of the earth. Assuming the mass of the earth to be concentrated at its Centre, the force of gravity or the weight of the body is given by

$$W = \frac{GMm}{R^2} = mg$$

$$\therefore \boxed{g = \frac{GM}{R^2}}$$

Mass and Density of the Earth

$$\text{Since, } g = \frac{GM}{R^2}$$

$$\text{Mass of the earth, } M = \frac{gR^2}{G}$$

Assuming the earth to be a homogeneous sphere,

$$\text{volume of the sphere} = \frac{4}{3} \pi R^3$$

$$\text{Density of the Earth} = D = \frac{\text{Mass}}{\text{volume}}$$

$$= \frac{gR^2/G}{\frac{4}{3} \pi R^3} = \frac{3g}{4\pi R G}$$

Variation in the acceleration due to gravity of the earth

The value of acceleration due to gravity on the surface of the earth,

$$g = \frac{GM}{R^2}$$

where 'G' and 'M' are constants

$$\text{Hence } \underline{g \propto \frac{1}{R^2}}$$

The polar radius is less than the equatorial radius of the earth. Therefore the value of g is more at the poles than at the equator.

I. Variation of 'g' with height

Let 'g' be the acceleration due to gravity on the surface of the earth and 'g'' be the acceleration due to gravity at a height 'h' above the earth's surface. Assuming earth to be a perfect sphere.

$$g = \frac{GM}{R^2} \text{ --- (1)}$$

$$g' = \frac{GM}{(R+h)^2} \text{ --- (2)}$$

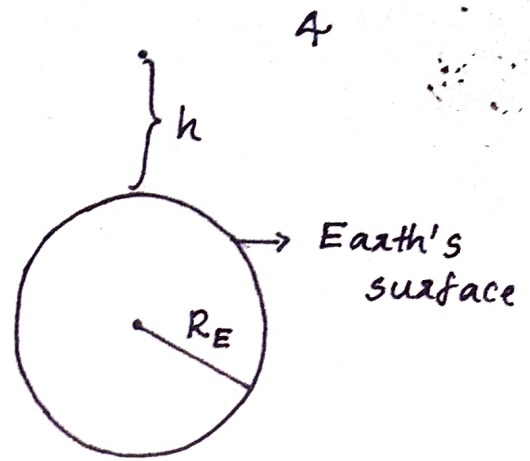
Dividing (1) by (2)

$$\frac{g}{g'} = \frac{\frac{GM}{R^2}}{\frac{GM}{(R+h)^2}} = \frac{(R+h)^2}{R^2}$$

$$\frac{g}{g'} = \left(1 + \frac{h}{R}\right)^2 \quad \therefore g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

$$g' = g \times \left(1 + \frac{h}{R}\right)^{-2}$$

$$g' = g \left(1 - \frac{2h}{R}\right)$$



II. Variation of 'g' with depth

Consider earth to be a homogeneous sphere of radius ' R ' and mass ' M ' with Centre O . If ' g ' is the acceleration due to gravity of the surface of the earth.

$$g = \frac{GM}{R^2} \text{ --- (1)}$$

but density $d, = \frac{\text{mass}}{\text{volume}}$

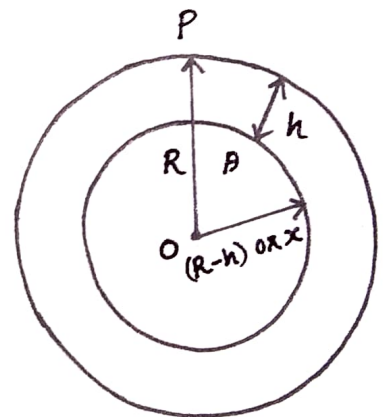
But volume of the sphere

$$V = \frac{4}{3} \pi R^3$$

$$M = d \cdot V$$

$$= \frac{4}{3} \pi R^3 \cdot d$$

$\therefore$ equation (1) becomes



$$g = \frac{G \cdot 4\pi R^3 \cdot d}{3 R^2}$$

$$g = \frac{4}{3} \pi G R d \quad \text{--- (2)}$$

Let ' g' ' be the acceleration due to gravity at a depth ' h ' below the surface of the earth then,

$$g' = \frac{G M'}{(R-h)^2}$$

$$= \frac{4}{3} \pi G (R-h) \cdot d \quad \text{--- (3)}$$

equation (3) by (2) gives

$$\frac{g'}{g} = \frac{\frac{4}{3} \pi G (R-h) d}{\frac{4}{3} \pi G R \cdot d}$$

$$\frac{g'}{g} = \frac{(R-h)}{R} = \left(1 - \frac{h}{R}\right)$$

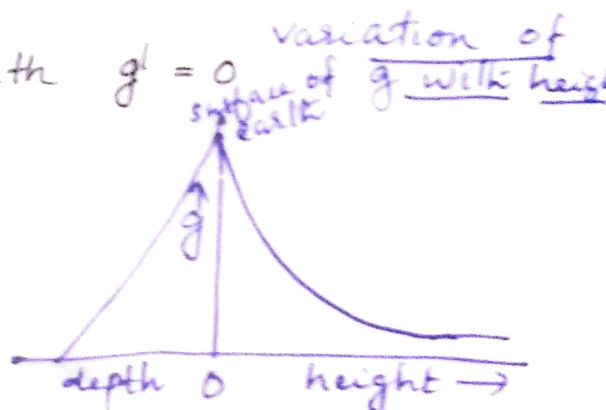
$$\therefore \boxed{g' = g \left(1 - \frac{h}{R}\right)}$$

At the Centre of the earth $g' = 0$ variation of g with height

$$\therefore \underline{h = R}$$

$$g' \propto (R-h)$$

$$g' \propto x$$



where $(R-h) = x$, the distance of the point from the centre of the earth.

$\therefore$ Acceleration due to gravity at any point inside the earth is proportional to the distance of the point from the Centre of the earth.

Gravitational field

The gravitational field of a body is the space around the body where its gravitational influence is felt.

Intensity of gravitational field (I)

The intensity of gravitational field is defined as the force experienced by a body of unit mass placed at that point.

$$\text{Intensity } I = \frac{F}{m}$$

The gravitational force on a mass 'm' placed near the surface of the earth is

$$F = \frac{GMm}{R^2}$$

$$\therefore I = \frac{\frac{GMm}{R^2}}{m} = \frac{GM}{R^2} \quad (\text{where } \frac{GM}{R^2} = g)$$

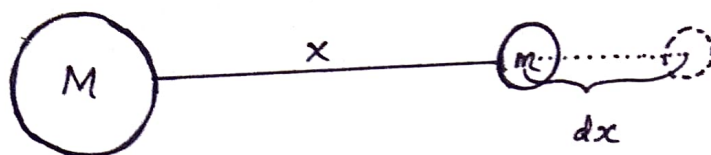
$$\therefore I = \frac{GM}{R^2}$$

$$I = g$$

Thus intensity of the gravitational field due to earth at a point on its surface is numerically equal to the acceleration due to gravity at the point.

Gravitational Potential energy

The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point.



consider a body of mass 'm' at a distance x from a large mass M . The gravitational force

$$F = \frac{GMm}{x^2}$$

The workdone in moving m . By a small distance dx along the direction of x .

$$\begin{aligned} \text{ie } dw &= Fx dx \\ &= \frac{GMm}{x^2} x dx \end{aligned}$$

The total work done in moving m from x_1 to x_2 from M . is obtained by integrating the equation with limits x_1 and x_2

Total workdone

$$\begin{aligned} W &= \int_{x_1}^{x_2} \frac{GMm}{x^2} \cdot dx \\ W &= GMm \int_{x_1}^{x_2} x^{-2} dx \\ W &= GMm \left[\frac{-1}{x} \right]_{x_1}^{x_2} \end{aligned} \quad \left| \begin{aligned} &\int_{x_1}^{x_2} x^{-2} dx \\ &= \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1} \\ &= -\frac{1}{x} \end{aligned} \right.$$

$$\text{ie } W = GMm \left[-\frac{1}{x_2} + \frac{1}{x_1} \right] = GMm \left[\frac{1}{x_1} - \frac{1}{x_2} \right]$$

In general the workdone remains as the increase in P.E, thus Potential Energy

$$\boxed{P.E = GMm \left[\frac{1}{x_1} - \frac{1}{x_2} \right]}$$

Potential Energy near the Surface of Earth

If the mass 'm' is lifted to a height 'h' from the earth's surface

$$r_1 = R \text{ and } r_2 = R+h \text{ Then,}$$

$$P.E = GMm \left[\frac{1}{R} - \frac{1}{(R+h)} \right]$$

$$= GMm \left[\frac{h}{R(R+h)} \right]$$

$$P.E = \frac{GMmh}{R(R+h)}$$

if $h \ll R$, then $R+h \approx R$ then,

$$P.E = \frac{GMm \times h}{R \times R} = \frac{GMmh}{R^2}$$

$$\text{But } \frac{GM}{R^2} = g \text{ then}$$

$$\boxed{P.E = mgh}$$

Gravitational Potential

The gain in gravitational P.E of a body of mass 'm' when it is raised from the surface of the earth to a distance r_1 is

$$P.E = GMm \left[\frac{1}{R} - \frac{1}{r_1} \right]$$

if r_1 and r_2 are the distances of two points from the earth,

$$P.E_1 = GMm \left[\frac{1}{R} - \frac{1}{r_1} \right] \text{ ——— (1)}$$

$$P.E_2 = GMm \left[\frac{1}{R} - \frac{1}{r_2} \right] \text{ ——— (2)}$$

The difference in P.E between these points

$$\begin{aligned}
 PE_1 - PE_2 &= GMm \left[\frac{1}{R} - \frac{1}{r_1} \right] - GMm \left[\frac{1}{R} - \frac{1}{r_2} \right] \\
 &= -GMm \cdot \frac{1}{r_1} + GMm \cdot \frac{1}{r_2} \\
 &= -GMm \left[\frac{1}{r_1} - \frac{1}{r_2} \right]
 \end{aligned}$$

If $r_2 = \text{infinity}$ then $\frac{1}{r_2} = 0$

$$\therefore PE_1 - PE_2 = -\frac{GMm}{r_1}$$

If r_1 is replaced by r , the potential energy at the ' r ' distance from centre of earth is

$$P.E = -\frac{GMm}{r}$$

For any finite potential Energy -ve. we can do work against gravity to increase potential energy. (to increase the separation between two masses)

If $m=1$ the P.E for unit mass is known as the Gravitational Potential (V) due to the mass ' m '.

$$\text{ie } V = -\frac{GM}{r}$$

The gravitational potential is the work done in bringing unit mass from infinity up to that point. The SI unit of gravitational potential is

$$J/kg$$

IMPORTANT PROBLEMS

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1. Calculate the mass of the earth. Radius of the earth is 6371 km and g is 9.8 m/s^2 .

$$R = 6371 \text{ km}$$

$$g = 9.8 \text{ m/s}^2$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$g = \frac{GM}{R^2}$$

$$M = \frac{gR^2}{G} = \frac{9.8 \times (6371)^2}{6.67 \times 10^{-11}} = 59636953 \times 10^{11} \text{ kg}$$
$$= \underline{\underline{59.64 \times 10^{17} \text{ kg}}}$$

2. At what height above the earth surface, the value of ' g ' same as in a mine 100 km deep?

At a height ' h ' above the surface of earth

$$g' = g \left[1 - \frac{2h}{R} \right] \text{ — (1)}$$

At a depth ' h ' below the earth's surface

$$g' = g \left(1 - \frac{h'}{R} \right) \text{ — (2)}$$

Equating (1) and (2)

$$g \left(1 - \frac{2h}{R} \right) = g \left(1 - \frac{h'}{R} \right)$$

$$\frac{2h}{R} = \frac{h'}{R}$$

$$2h = h' \quad \therefore 2h = 100 \text{ km}$$

$$\therefore h = \frac{100}{2} = \underline{\underline{50 \text{ km}}}$$

3. Acceleration due to gravity at moon surface

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is 1.67 m/s^2 . If the radius of the moon is $1.74 \times 10^6 \text{ m}$, calculate the mass of the moon

We have acceleration due to gravity

$$g = \frac{GM}{R^2}$$

$$g = 1.67 \text{ m/s}^2$$

$$R = 1.74 \times 10^6 \text{ m}$$

$$M = \frac{gR^2}{G} = \frac{1.67 \times (1.74 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$= \frac{1.67 \times 1.74 \times 1.74}{6.67} \times 10^{23} = 0.75803 \times 10^{23}$$

$$= \underline{\underline{7.58 \times 10^{22} \text{ kg}}}$$

4. How much below the surface does the g become 70% of the value on the surface of the earth $R = 6.4 \times 10^6 \text{ m}$.

Acceleration due to gravity below the earth surface

$$g' = g \left(1 - \frac{h}{R}\right)$$

$$\frac{g'}{g} = 1 - \frac{h}{R} \quad \text{--- (1)}$$

From the data given

$$g' = 70\% \times g$$

$$\frac{g'}{g} = \frac{70}{100} \quad \text{--- (2)} \quad \text{Equating (1) and (2)}$$

$$\frac{1-h}{R} = \frac{70}{100} = \frac{1-h}{R} = \frac{7}{10}$$

$$\frac{1-7}{10} = \frac{h}{R} \Rightarrow \frac{10-7}{10} = \frac{h}{R}$$

$$h = \frac{R \times 3}{10} = \frac{6.4 \times 10^6 \times 3}{10} = \underline{\underline{1.92 \times 10^6 \text{ m}}}$$

1000 5.

A body weights 63 N on the surface of the earth. What is the gravitational force on it at a height equal to half the radius of the earth?

At the surface of the earth,

$$g = \frac{GM}{R^2} \text{ --- (1)}$$

At a height equal to half of radius of earth

$$g' = \frac{GM}{(R + \frac{R}{2})^2} \text{ --- (2)}$$

Dividing (2) by (1) we get

$$\frac{g'}{g} = \frac{\frac{GM}{(R + \frac{R}{2})^2}}{\frac{GM}{R^2}} = \frac{R^2}{(R + \frac{R}{2})^2} = \frac{R^2}{(\frac{3R}{2})^2}$$

$$= (R \times \frac{2}{3R})^2 = (\frac{2}{3})^2$$

$$g' = g (\frac{2}{3})^2 = g \frac{4}{9}$$

Gravitational force at a height

$$\begin{aligned} Mg' &= M \times g \times \frac{4}{9} \\ &= 63 \times \frac{4}{9} = \underline{\underline{28 \text{ N}}} \end{aligned}$$

6. The acceleration due to gravity on the moon surface is 1.7 m/s^2 . The radius of the moon is 0.27 times that of earth. 'g' on the earth surface = 9.8 m/s^2 . Find the ratio of the mass of the earth to that of moon.

Acceleration due to gravity on earth surface

$$g_e = \frac{GM_e}{R_e^2} \text{ --- (1)}$$

on the moon surface

$$g_m = \frac{G M_m}{R_m^2} \quad \text{--- (2)}$$

dividing (1) by (2)

$$\frac{g_e}{g_m} = \frac{M_e}{R_e^2} \times \frac{R_m^2}{M_m}$$

$$\frac{g_e}{g_m} = \frac{M_e}{M_m} \times \left(\frac{R_m}{R_e}\right)^2$$

$$\frac{M_e}{M_m} = \frac{g_e}{g_m} \times \left(\frac{R_m}{R_e}\right)^2 = \frac{9.8}{1.7} \times \left(\frac{R_e}{R_e \times 21}\right)^2$$

$$\frac{M_e}{M_m} = \frac{9.8}{1.7} \times \frac{1}{(21)^2} = \underline{\underline{79.077}}$$

Satellite

A satellite is a body which is constantly revolving in an orbit around a planet. The moon is the natural satellite of the earth. It revolves around the earth once in 27.3 days.

Artificial Satellites

It is a manmade object placed at a height above the earth and given sufficient velocity so as to revolve around the earth in closed orbit.

Orbital Velocity

The velocity with which a satellite moves in its closed orbit is called orbital velocity.

Consider a satellite of mass 'm' moving around in a closed orbit of radius 'r'

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with orbital velocity ' v '. Let M be the mass of the earth and ' R ' its radius when the satellite is in its stable orbit the centripetal force is provided by gravitational force.

$$\text{ie } \frac{Mv^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

If ' h ' is the height of the satellite above the earth

$$r = (R+h)$$

$$v = \sqrt{\frac{GM}{(R+h)}}$$

$$\text{But } g = \frac{GM}{R^2} \therefore GM = gR^2$$

If the orbit is close to the earth

$$h \ll R$$

$$\therefore r \approx R$$

$$\therefore (R+h) \approx R$$

$$\therefore v = \sqrt{\frac{gR^2}{R}} = \sqrt{gR}$$

$$\therefore v = \sqrt{gR}$$

This orbit is called minimum orbit.
The velocity corresponding to minimum orbit is called first Cosmic Velocity

Impo: First Cosmic velocity $v = \sqrt{Rg} = \underline{\underline{7.92 \text{ km/s}}}$

Time period of a Satellite

The time period of a satellite is the time taken by it to revolve once around the earth.

$$T = \frac{\text{circumference of orbit}}{\text{orbital velocity}}$$

$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi r}{\sqrt{GM/r}}$$

$$T = 2\pi \sqrt{\frac{r^3}{GM}}$$

$$T = 2\pi \left(\frac{r^3}{GM} \right)^{1/2}$$

$$= 2\pi \sqrt{\frac{r^3}{GM}}$$

For a satellite revolving around the earth at a height 'h' from the surface of the earth

$$r = R + h$$

$$\therefore r^3 = (R + h)^3$$

$$\therefore \boxed{T = 2\pi \sqrt{\frac{(R+h)^3}{GM}}}$$

$$g = \frac{GM}{R^2} \therefore GM = gR^2$$

$$\therefore \boxed{T = 2\pi \sqrt{\frac{(R+h)^3}{gR^2}}}$$

For a satellite of minimum orbit i.e. $r = R$

$$\therefore R + h = r = R$$

$$\therefore \boxed{T = 2\pi \sqrt{\frac{R^3}{GM}}}$$

$$GM = gR^2$$

$$\therefore \boxed{T = 2\pi \sqrt{\frac{R^3}{gR^2}}}$$

$$\therefore \boxed{T = 2\pi \sqrt{\frac{R}{g}}}$$

Geostationary Satellites

(Geosynchronous Satellite)

They are artificial satellites orbiting the earth so that their period is synchronous with (same) that of the earth. The orbit in which they are staying is parking orbit (geostationary orbit). Such satellites must move in circular orbits in the equatorial plane of the earth and in the same direction as that of the rotation of earth. [west $\rightarrow$ east].

The period of synchronous satellite $T = 24$ hrs.

Such satellites are used for communication purpose and are called as SYNCOMS (Synchronous Communication Satellites). Radio, T.V, and telephone signals are directed to a satellite from the station on earth. The satellite retransmit them to some other stations on the earth. In this way the range of communication has been increased. Accurate and precise forecasting can be made with the help of them.

Polar Satellites

Polar orbiting satellites orbit closely parallel to the earth's median lines. They pass over the north and south poles in each revolution.

Each orbit takes to 100 minutes, resulting in about 14 orbits a day. The satellite covers the entire surface of the earth twice each day.

Total Energy of an Orbiting Satellite

Let 'M' be the mass of the earth and 'm' be the mass of the satellite 'r' be the radius of the orbit of satellite and 'v' be the velocity of satellite.

$$\text{P.E of the satellite} = -\frac{G M m}{r}$$

$$\text{K.E of the satellite} = \frac{1}{2} m v^2$$

From the law of gravitation

$$\frac{m v^2}{r} = \frac{G M m}{r^2}$$

$$v^2 = \frac{G M}{r}$$

$$\text{KE} = \frac{1}{2} m \times \frac{G M}{r} = \frac{G M m}{2r}$$

$$\begin{aligned} \text{Total Energy} &= \text{P.E} + \text{K.E} \\ &= -\frac{G M m}{r} + \frac{G M m}{2r} \\ &= \frac{G M m}{r} \left(\frac{1}{2} - 1 \right) \\ &= -\frac{G M m}{2r} \end{aligned}$$

ie Total Energy = $-\frac{G M m}{2r}$

Binding Energy = $+\frac{G M m}{2r}$

Escape velocity (escape speed)

The minimum velocity with which a body must be projected so that it may escape from the gravitational field of the earth is called escape velocity from the earth.

Let 'M' be the mass of the earth and 'R' its radius. Let v_e be the velocity of the body of mass 'm' with which it is projected so that it escapes the gravitational field of the earth.

K.E of the body near the surface
 $= \frac{1}{2} m v_e^2$

P.E of the body on the surface of earth
 $= -\frac{G M m}{R}$

Total Energy of the body near the surface
 $= \frac{1}{2} m v_e^2 - \frac{G M m}{R}$

At infinite distance, both KE and PE are zero. Applying the law of Conservation of energy.

$$\frac{1}{2} m v_e^2 - \frac{G M m}{R} = 0$$

$$\frac{1}{2} m v_e^2 = \frac{G M m}{R}$$

$$v_e^2 = \frac{2GM}{R}$$

$$v_e = \sqrt{\frac{2GM}{R}}$$

But $g = \frac{GM}{R^2} \therefore GM = gR^2$

$$\therefore v_e = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR}$$

$$\boxed{v_e = \sqrt{2gR}}$$

we have orbital velocity $v_o = \sqrt{gR}$

$$\therefore \underline{v_e = \sqrt{2} v_o}$$

escape velocity is $\sqrt{2}$ times orbital velocity

The escape velocity of a body on earth is called Second Cosmic velocity.

$$\text{Second Cosmic velocity} = \underline{\underline{11.2 \text{ km/sec}}}$$

KEPLER'S LAW

1. Law of elliptical orbit (1st law)

Every planet moves around the sun in an elliptical orbit with sun at one of its foci.

2. Law of equal areas (2nd law)

The line joining the planet to the sun sweeps out equal areas in equal intervals of time.

or the areal velocity is a constant.

$$\text{ie, } \frac{dA}{dt} = \text{a Constant.}$$

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3. Harmonic Law (3rd law)

The square of the period of revolution of a planet around the sun is proportional to the cube of the semi major axis of its orbit.

$$\text{ie, } T^2 \propto a^3$$

where 'T' is the period of the planet and 'a' is the semi major axis of its orbit.

Important Problems

1. There is practically no atmosphere on the moon. Give reason?

The escape velocity on the moon being very small, the air molecules have escaped from the surface of the moon.

2. Most of the Hydrogen molecules are supposed to have escaped from earth's atmosphere. Give reason.

Light molecules like hydrogen will easily acquire a velocity greater than 11.2 km/sec due to thermal fluctuation and will escape from the earth's atmosphere.

3. Can pendulum clock be used in an artificial satellite? explain.

No, since weight $w=0$, $g=0$

$$\therefore T = \infty \therefore \left[T = 2\pi \sqrt{\frac{l}{g}} \right]$$

The pendulum will not oscillate and clock will not run.

4. An elephant and an ant are to be projected out of earth into space. Do we acquire different velocities to do so?

No. $v_e = \sqrt{2gR}$ it is independent of the mass of the body.

5. The satellite does not require any fuel to encircle the earth. why?

Centripetal force for the satellite in its orbit while rotating is provided by the gravitational force of the earth on the satellite

ie, $\left(\frac{GMm}{r^2} \right) \therefore$ No fuel is needed.

6. A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun. If the earth is 1.5×10^8 km away from the sun.

By Kepler's law

$$T^2 \propto R^3$$

$$T_e^2 \propto R_e^3 \text{ — (1)}$$

$$T_s^2 \propto R_s^3 \text{ — (2)}$$

(1) $\div$ (2) gives

$$\left(\frac{T_e}{T_s} \right)^2 = \left(\frac{R_e}{R_s} \right)^3$$

Here $T_e = 1 \text{ year}$

$$T_s = 29.5 \text{ years}$$

$$R_e = 1.5 \times 10^8 \text{ km}$$

$$\begin{aligned}
 \therefore R_s^2 &= \left(\frac{T_s}{T_e}\right)^2 \times R_e^3 \\
 &= (29.5)^2 \times (1.5 \times 10^8)^3 \\
 &= 870.25 \times 3.375 \times 10^{24} \\
 &= 2931.094 \times 10^{24} \\
 &= 2.9311 \times 10^{27} \\
 R_s &= \sqrt[3]{2.9311 \times 10^{27}} = 1.432 \times 10^9 \\
 &= \underline{\underline{14.3 \times 10^8 \text{ km}}}
 \end{aligned}$$

7. Jupiter has a mass 318 times that of the earth and its radius is 11.2 times the earth's radius. Estimate the escape velocity of the body from the Jupiter surface? Escape velocity from the earth = 11.2 km/sec. Does its atmosphere contain light gases?

We have escape velocity of the earth

$$= \sqrt{\frac{2GM_e}{R_e}}$$

Escape velocity from the Jupiter surface

$$V_j = \sqrt{\frac{2GM_j}{R_j}}$$

Here

$$M_j = 318 M_e$$

$$R_j = 11.2 R_e$$

$$\therefore V_j = \sqrt{\frac{2G \times 318 M_e}{11.2 R_e}}$$

$$V_j = \sqrt{\frac{2GM_e}{R_e}} \times \sqrt{\frac{318}{11.2}}$$

$$\begin{aligned}
 V_j &= V_e \times \sqrt{\frac{318}{11.2}} = 11.2 \times 5.3285 \\
 &= \underline{\underline{59.679 \text{ m/s}}}
 \end{aligned}$$

Escape velocity on Jupiter is very high. $\therefore$ Light molecules cannot escape. They will be present in Jupiter's atmosphere.

8. Determine the escape velocity of a body from the moon. Take the moon to be a uniform sphere of radius $1.74 \times 10^6 \text{ m}$ and mass $7.36 \times 10^{22} \text{ kg}$

$$\text{Escape Velocity } V_m = \sqrt{\frac{2GM}{R}}$$

$$R = 1.74 \times 10^6 \text{ m} ; M = 7.36 \times 10^{22} \text{ kg}$$

$$V_m = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.36 \times 10^{22}}{1.74 \times 10^6}}$$

$$= \sqrt{56.42667 \times 10^5} = \sqrt{5.642667 \times 10^6}$$

$$= \underline{\underline{2.375 \times 10^3 \text{ m/s}}}$$

Important Points

- * A uniform spherical shell of matter attracts a particle that is outside the shell as if the shell's mass were concentrated at its centre
- * Gravitation and principle of superposition

$$\begin{aligned} \vec{F}_1 \text{ net} &= \vec{F}_{1-2} + \vec{F}_{1-3} + \dots + \vec{F}_{1-n} \quad [\text{vector sum}] \\ &= \sum_{i=1}^n \vec{F}_{1-i} \end{aligned}$$

$\vec{F}_i = \int d\vec{F}$ - For gravitational force from an extended body.

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* A uniform shell of matter exerts no net gravitational force on a particle located inside it.

* $U \propto$ - P.E at ∞ U - P.E at a point

$$U_{\infty} - U = -W$$

work done to take the object from a point to infinity

$$= -\left(-\frac{GMm}{r}\right) = \frac{GMm}{r}$$